Natural Language Processing

Language modeling

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Announcements

- Quiz 3 potential topics:
  - Logistic Regression: feature extraction, classification decision, loss, optimization with SGD, regularization
  - HW1 questions
- HW1 due: Oct 30
- HW2 out: Oct 30
- Nov 1 class in a different room

The Language Modeling problem

- Assign a probability to every sentence (or any string of words)
  - finite vocabulary (e.g. words or characters)
  - infinite set of sequences

$$\sum_{\mathbf{e}\in\Sigma^*} p_{\mathrm{LM}}(\mathbf{e}) = 1$$

$$p_{\mathrm{LM}}(\mathbf{e}) \geq 0 \quad \forall \mathbf{e} \in \Sigma^*$$

The Language Modeling problem

- Assign a probability to every sentence (or any string of words)
  - finite vocabulary (e.g. words or characters) *{the, a, telescope, …}*
  - infinite set of sequences
    - *a telescope STOP*
    - *a STOP*
    - *the the the STOP*
    - *I saw a woman with a telescope STOP*
    - *STOP*
    - *...*

Language models play the role of ...

- a judge of grammaticality
- a judge of semantic plausibility
- an enforcer of stylistic consistency
- a repository of knowledge (?)

p(*disseminating so much currency STOP*) = 10^{-15}

p(*spending a lot of money STOP*) = 10^{-9}

Equivalent definition

- **Language Modeling** is the task of predicting what word comes next

the students opened their ______

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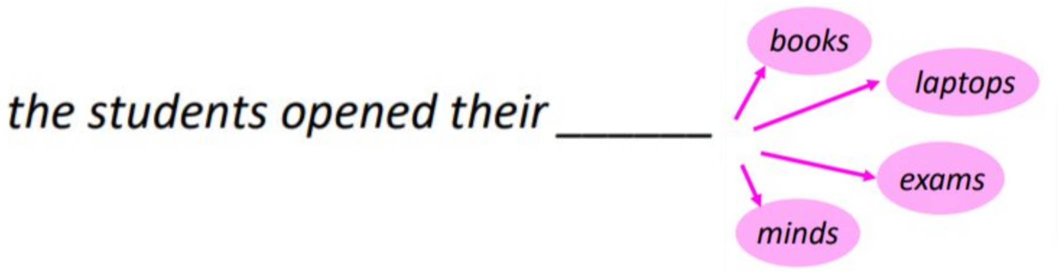

- More formally: given a sequence of words $x^{(1)}, x^{(2)}, \ldots x^{(t)}$
compute the probability distribution if the next word $x^{(t+1)}$
Where $x^{(t+1)}$ can be any word in the vocabulary $V=\{ w_1, w_2, \ldots w_{|V|}\}$

Language Modeling

- If we have some text, then the probability of this text (according to the Language Model) is:

$$P(\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}) = P(\boldsymbol{x}^{(1)}) \times P(\boldsymbol{x}^{(2)} | \boldsymbol{x}^{(1)}) \times \cdots \times P(\boldsymbol{x}^{(T)} | \boldsymbol{x}^{(T-1)}, \ldots, \boldsymbol{x}^{(1)})$$

$$= \prod_{t=1}^{T} \underbrace{P(\boldsymbol{x}^{(t)} | \boldsymbol{x}^{(t-1)}, \ldots, \boldsymbol{x}^{(1)})}$$

This is what our LM provides

n-gram Language Models

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

- Question: How to learn a Language Model?

A trivial model

- Assume we have n training sentences
- Let $x_1, x_2, \ldots, x_n$ be a sentence, and $c(x_1, x_2, \ldots, x_n)$ be the number of times it appeared in the training data.
- Define a language model:

$$p(x_1, \ldots, x_n) = \frac{c(x_1, \ldots, x_n)}{N}$$

- No generalization!

n-gram Language Models

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- Question: How to learn a Language Model?
- Answer (pre- Deep Learning): learn an *n-gram* Language Model!

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- Definition: An n-gram is a chunk of n consecutive words.

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  - trigrams: {I have a, have a dog, a dog whose, … , playing with Lucy}

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  - bigrams: {I have, have a, a dog, dog whose, … , with Lucy}
  - trigrams: {I have a, have a dog, a dog whose, … , playing with Lucy}
  - four-grams: {I have a dog, … , like playing with Lucy}
  - …

n-gram Language Models

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

- w_1 – a unigram
- $w_1\ w_2$ – a bigram
- $w_1\ w_2\ w_3$ – a trigram
- $w_1 w_2 \ldots w_n$ – an n-gram

n-gram Language Models

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

- Question: How to learn a Language Model?
- Answer (pre- Deep Learning): learn an *n-gram* Language Model!

- Idea: Collect statistics about how frequent different n-grams are and use these to predict next word

unigram probability

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

- corpus size m = 17
- P(Lucy) = 2/17; P(cats) = 1/17

- Unigram probability: $P(w) = \frac{\text{count}(w)}{m} = \frac{C(w)}{m}$

bigram probability

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

$$P(A \mid B) = \frac{P(A,B)}{P(B)}$$

$$P(\text{have} \mid \text{I}) = \frac{C\,(\text{I have})}{C\,(\text{I})} = \frac{2}{2} = 1$$

$$P(\text{two} \mid \text{have}) = \frac{C\,(\text{have two})}{C\,(\text{have})} = \frac{1}{2} = 0.5$$

$$P(\text{eating} \mid \text{have}) = \frac{C\,(\text{have eating})}{C\,(\text{have})} = \frac{0}{2} = 0$$

$$P(w_2 \mid w_1) = \frac{C(w_1, w_2)}{\sum_w C(w_1, w)} = \frac{C(w_1, w_2)}{C(w_1)}$$

trigram probability

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

$$P(A \mid B) = \frac{P(A,B)}{P(B)}$$

$$P(a \mid I\ have) = \frac{C(I\ have\ a)}{C(I\ have)} = \frac{1}{2} = 0.5$$

$$P(several \mid I\ have) = \frac{C(I\ have\ several)}{C(I\ have)} = \frac{0}{2} = 0$$

$$P(w_3 | w_1\, w_2) = \frac{C(w_1, w_2, w_3)}{\sum_w C(w_1, w_2, w)} = \frac{C(w_1, w_2, w_3)}{C(w_1, w_2)}$$

n-gram probability

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

$$P(A \mid B) = \frac{P(A,B)}{P(B)}$$

$$P(w_i \mid w_1, w_2, \ldots, w_{i-1}) = \frac{C(w_1, w_2, \ldots, w_{i-1}, w_i)}{C(w_1, w_2, \ldots, w_{i-1})}$$

Sentence/paragraph/book probability

$$P(\boldsymbol{x}^{(1)},\ldots,\boldsymbol{x}^{(T)}) = P(\boldsymbol{x}^{(1)}) \times P(\boldsymbol{x}^{(2)}|\,\boldsymbol{x}^{(1)}) \times \cdots \times P(\boldsymbol{x}^{(T)}|\,\boldsymbol{x}^{(T-1)},\ldots,\boldsymbol{x}^{(1)})$$

$$= \prod_{t=1}^{T} P(\boldsymbol{x}^{(t)}|\,\boldsymbol{x}^{(t-1)},\ldots,\boldsymbol{x}^{(1)})$$

P(its water is so transparent that the) =

P(its) ×
P(water | its) ×
P(is | its water) ×
P(so | its water is) ×
P(transparent | its water is so) ×
… ×

P(the | its water is so transparent that) → How to estimate?

Markov assumption


Andrei Markov

- We make the Markov assumption: $x^{(t+1)}$ depends only on the preceding $n-1$ words
  - Markov chain is a "*…stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event.*"

$$P(\boldsymbol{x}^{(t+1)}|\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(1)}) = P(\boldsymbol{x}^{(t+1)}|\underbrace{\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)}}_{\text{n-1 words}})$$

assumption

Markov assumption


Andrei Markov

P(the | its water is so transparent that) ≈ P(the | transparent that)

or maybe even

P(the | its water is so transparent that) ≈ P(the | that)

Calculating a probability of a sequence

Chain rule

$$p(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) =$$

$$p(X_1 = x_1) \prod_{i=2}^{n} p(X_i = x_i \mid X_1 = x_1, \ldots, X_{i-1} = x_{i-1})$$

First-order Markov process

Chain rule

$$p(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) =$$
$$p(X_1 = x_1) \prod_{i=2}^{n} p(X_i = x_i \mid X_1 = x_1, \ldots, X_{i-1} = x_{i-1})$$

Markov assumption

$$= \quad P(X_1 = x_1) \prod_{i=2}^{n} P(X_i = x_i | X_{i-1} = x_{i-1})$$

Second-order Markov process:

- Relax independence assumption:

$$p(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) =$$
$$p(X_1 = x_1) \times p(X_2 = x_2 \mid X_1 = x_1)$$
$$\times \prod_{i=3}^{n} p(X_i = x_i \mid X_{i-2} = x_{i-2}, X_{i-1} = x_{i-1})$$

Second-order Markov process:

- Relax independence assumption:

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- Simplify notation:

$$x_0 = *, x_{-1} = *$$

3-gram LMs

- A trigram language model contains
  - a vocabulary V
  - a non negative parameters q(*w*|*u*,*v*) for every trigram, such that

$$w \in \mathcal{V} \cup \{\text{STOP}\},\ \ u, v \in \mathcal{V} \cup \{*\}$$

  - the probability of a sentence $x_1, \ldots, x_n$, where x_n=STOP is

$$p(x_1, \ldots, x_n) = \prod_{i=1}^{n} q(x_i \mid x_{i-1}, x_{i-2})$$

Example

$$p(\text{the dog barks STOP}) =$$

Example

$$p(\text{the dog barks STOP}) = q(the \mid *, *) \times$$

Example

$$
\begin{aligned}
p(\text{the dog barks STOP}) = & q(the \mid *, *) \times \\
& q(dog \mid *, the) \times \\
& q(barks \mid the, dog) \times \\
& q(STOP \mid dog, barks) \times
\end{aligned}
$$

Berkeley restaurant project sentences

- can you tell me about any good cantonese restaurants close by
- mid priced that food is what i'm looking for
- tell me about chez pansies
- can you give me a listing of the kinds of food that are available
- i'm looking for a good place to eat breakfast
- when is caffe venezia open during the day

Raw bigram counts (~1000 sentences)

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Bigram probabilities

$$P(w_i \mid w_{i-1}) = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}$$

$$P(w_1, w_2, \ldots, w_n) \approx \prod_i P(w_i \mid w_{i-1})$$

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Bigram estimates of sentence probability

$$P(w_i \mid w_{i-1}) = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}$$

$$P(w_1, w_2, \ldots, w_n) \approx \prod_i P(w_i \mid w_{i-1})$$

P(<s> i want chinese food </s>) =

P(i|<s>)

× P(want|i)

× P(chinese|want)

× P(food|chinese)

× P(</s>|food)

= …

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
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lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

What can we learn from bigram estimates?

P(to|want) = 0.66

P(chinese|want) = 0.0065

P(eat|to) = 0.28

P (i|<s>) = 0.25

P(food|to) = 0.0

P(want|spend) = 0.0

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
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lunch	0.0059	0	0	0	0	0.0029	0	0
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Sampling from a language model

1 gram	Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives

Sampling from a language model

1 gram	Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives
2 gram	Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her

Sampling from a language model

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2 gram	Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her
3 gram	They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

Sampling from a language model

1 gram	–To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have –Hill he late speaks; or! a more to leg less first you enter
2 gram	–Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow. –What means, sir. I confess she? then all sorts, he is trim, captain.
3 gram	–Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done. –This shall forbid it should be branded, if renown made it empty.
4 gram	–King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in; –It cannot be but so.

Practical issues

- Multiplying very small numbers results in numerical underflow
  - we do every operation in log space
  - (also adding is faster than multiplying)

Markovian assumption is false

He is from France, so it makes sense that his first language is…

- We would want to model longer dependencies

Sparsity

- Maximum likelihood for estimating q
  - Let $c(w_1, \ldots, w_n)$ be the number of times that *n*-gram appears in a corpus

$$q(w_i \mid w_{i-2}, w_{i-1}) = \frac{c(w_{i-2}, w_{i-1}, w_i)}{c(w_{i-2}, w_{i-1})}$$

  - If vocabulary has 20,000 words ⇒ Number of parameters is 8×10^{12}!

Bias-variance tradeoff

- Given a corpus of length M

Trigram model:

$$q(w_i \mid w_{i-2}, w_{i-1}) = \frac{c(w_{i-2}, w_{i-1}, w_i)}{c(w_{i-1}, w_i)}$$

Bigram model:

$$q(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

Unigram model:

$$q(w_i) = \frac{c(w_i)}{M}$$

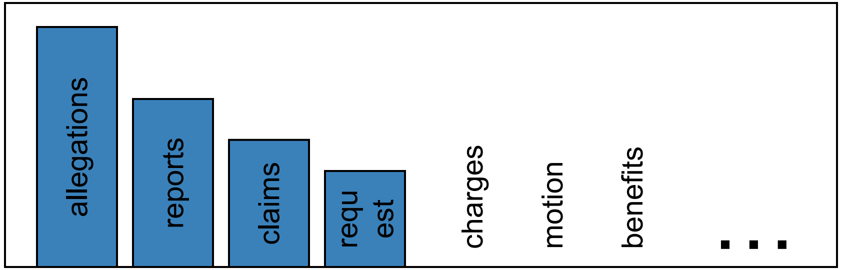
Dealing with sparsity

- For most N-grams, we have few observations
- General approach: modify observed counts to improve estimates
  - Back-off:
    - use trigram if you have good evidence;
    - otherwise bigram, otherwise unigram
  - Interpolation: approximate counts of N-gram using combination of estimates from related denser histories
  - Discounting: allocate probability mass for unobserved events by discounting counts for observed events

Discounting/smoothing methods

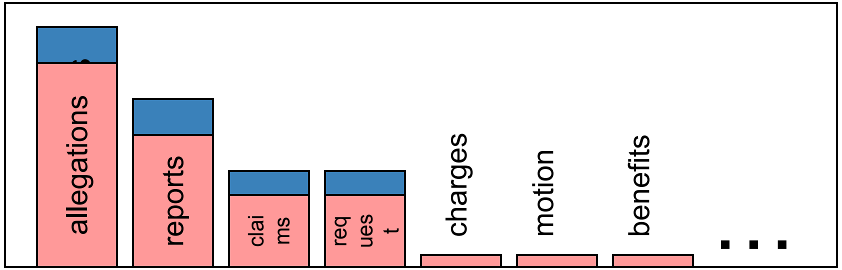
- We often want to make estimates from sparse statistics:

P(w | denied the)
3 allegations
2 reports
1 claims
1 request
7 total



- Smoothing flattens spiky distributions so they generalize better:

P(w | denied the)
2.5 allegations
1.5 reports
0.5 claims
0.5 request
2 other
7 total



Linear interpolation

- Combine the three models to get all benefits

$$
\begin{aligned}
q_{LI}(w_i \mid w_{i-2}, w_{i-1}) = & \lambda_1 \times q(w_i \mid w_{i-2}, w_{i-1}) \\
& + \lambda_2 \times q(w_i \mid w_{i-1}) \\
& + \lambda_3 \times q(w_i)
\end{aligned}
$$

$$
\lambda_i \geq 0, \;\; \lambda_1 + \lambda_2 + \lambda_3 = 1
$$

Linear interpolation

- Need to verify the parameters define a probability distribution

$$\sum_{w \in \mathcal{V}} q_{LI}(w \mid u, v)$$

$$= \sum_{w \in \mathcal{V}} \lambda_1 \times q(w \mid u, v) + \lambda_2 \times q(w \mid v) + \lambda_3 \times q(w)$$

$$= \lambda_1 \sum_{w \in \mathcal{V}} q(w \mid u, v) + \lambda_2 \sum_{w \in \mathcal{V}} q(w \mid v) + \lambda_3 \sum_{w \in \mathcal{V}} \times q(w)$$

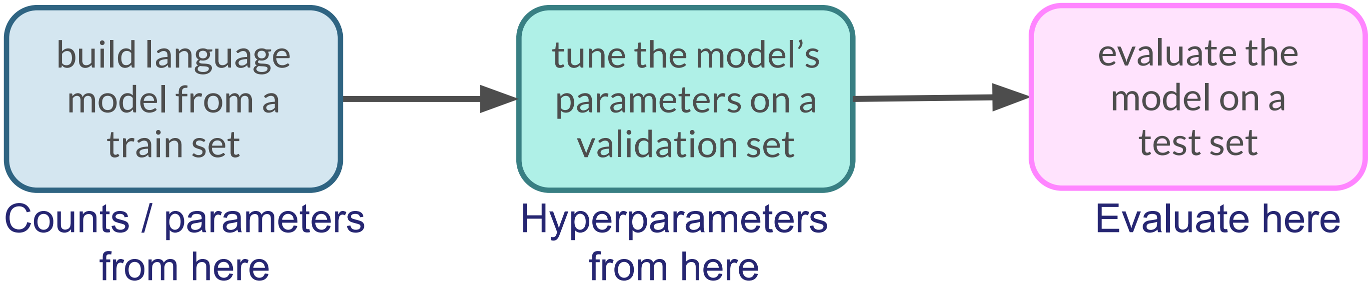
$$= \lambda_1 + \lambda_2 + \lambda_3 = 1$$

Dealing with Out-of-vocabulary terms

- Define a special OOV or “unknown” symbol `<unk>`. Transform some (or all) rare words in the training data to `<unk>`
  - You cannot fairly compare two language models that apply different <unk> treatments
- Build a language model at the character level

Evaluation

- Intuitively, language models should assign high probability to real language they have not seen before
  - Want to maximize likelihood on held-out, not training data
  - Models derived from counts / sufficient statistics require generalization parameters to be tuned on held-out data to simulate test generalization
  - Set hyperparameters to maximize the likelihood of the held-out data (usually with grid search or EM)



Evaluation

- Extrinsic evaluation: build a new language model, use it for some task (MT, ASR, etc.)
- Intrinsic: measure how good we are at modeling language

Extrinsic evaluation of N-gram models

- Best evaluation for comparing models A and B
  - Put each model in a task
    - spelling corrector, speech recognizer, MT system
  - Run the task, get an accuracy for A and for B
    - How many misspelled words corrected properly
    - How many words translated correctly
- Compare accuracy for A and B

Difficulty of extrinsic (in-vivo) evaluation of N-gram models

- Extrinsic evaluation
  - Time-consuming; can take days or weeks

So

- Sometimes use intrinsic evaluation: perplexity
  - Bad approximation
    - unless the test data looks just like the training data
  - So generally only useful in pilot experiments
  - But is helpful to think about

Intrinsic evaluation: perplexity

- Test data: $S = \{s_1, s_2, \ldots, s_{sent}\}$
  - parameters are estimated on training data

$$p(\mathcal{S}) = \prod_{i=1}^{sent} p(s_i)$$

  - *sent* is the number of sentences in the test data

Evaluation: perplexity

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  - parameters are estimated on training data

$$p(S) = \prod_{i=1}^{sent} p(s_i)$$

$$\begin{aligned} p(\text{the dog barks STOP}) = & q(the \mid *, *)\times \\ & q(dog \mid *, the)\times \\ & q(barks \mid the, dog)\times \\ & q(STOP \mid dog, barks)\times \end{aligned}$$

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$$\log_2 p(\mathcal{S}) = \sum_{i=1}^{sent} \log_2 p(s_i)$$

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$$\text{perplexity} = 2^{-l}, \; l = \frac{1}{M} \sum_{i=1}^{sent} \log_2 p(s_i)$$

  - *sent* is the number of sentences in the test data
  - M is the number of words in the test corpus

Evaluation: perplexity

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$$\text{perplexity} = 2^{-l}, \; l = \frac{1}{M} \sum_{i=1}^{sent} \log_2 p(s_i)$$

  - *sent* is the number of sentences in the test data
  - M is the number of words in the test corpus
  - A good language model has high p(S) and low perplexity

Understanding perplexity

$$\text{perplexity} = 2^{-\frac{1}{M}\sum_{i=1}^{sent} \log_2 p(s_i)}$$

- It's a branching factor
  - assign probability of 1 to the test data ⇒ perplexity = 1
  - assign probability of 1/|V| to every word ⇒ perplexity = |V|
  - assign probability of 0 to anything ⇒ perplexity = ∞
    - this motivates the proper probability constraint

$$\sum_{\mathbf{e} \in \Sigma^*} p_{\text{LM}}(\mathbf{e}) = 1$$

$$p_{\text{LM}}(\mathbf{e}) \geq 0 \quad \forall \mathbf{e} \in \Sigma^*$$

- cannot compare perplexities of LMs trained on different corpora

Typical values of perplexity

- When |V| = 50,000
- trigram model perplexity: 74 (<< 50,000)
- bigram model: 137
- unigram model: 955